Design and Testing of SMA Temperature-Compensated Cavity Resonators

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***Abstract*—In this paper, we present a method for designing temperature-compensated cavity resonators using shape memory alloys (SMAs). This paper gives an expression for the temperature drift of resonant frequency, which is valid for any conductor-loaded cavity regardless of its shape. This formula, combined with a field perturbation model, is used to derive the resonant frequency of an SMA-compensated resonator subject to temperature fluctuation. Experimental results are given that confirm the feasibility of the proposed design approach. A design method is proposed for specifying SMA alloys to build the actuator. An expression is derived to accurately predict the performance of an actuator design.**

***Index Terms*—Cavity resonator, filter, temperature compensation.**

I. Introduction

TEMPERATURE drift is an important consideration when designing RF filters. In most applications, the filters are required to meet RF performance specifications over a wide temperature range (−20 °C to 75 °C). This problem is typically circumvented by either having enough error margin in the design to accommodate any changes in the filter performance over temperature, or by constructing resonators from materials with low coefficients of thermal expansion (CTEs) such as invar. Satellite and today's wireless communication systems demand an efficient utilization of the assigned frequency spectrum, reducing the allowable design margin.

While invar has a very low CTE, it has high density, is difficult to machine, and has low thermal conductivity. Other materials such as aluminum have more favorable properties, but cannot be used in most applications because their higher CTE leads to unacceptable temperature drift. Over the past five years, efforts have been reported for temperature compensation of cavity resonators [1]–[4]. Most of these efforts focus on using combinations of materials in constructing the resonators.

In this paper, we propose using a spring-biased shape memory alloy (SMA) actuator to adjust the length of a tuning rod inside a resonant cavity. The unique properties of SMAs can be used to construct a tuning-rod actuator that will provide a temperature-dependent field perturbation in such a way as to compensate for temperature drift due to volumetric expansion. An SMA's ability to provide significant temperature-actuated deflection is unique. The deflection provided by an SMA subject to temperature increase is much larger than other schemes (e.g., bi-metals).

This actuator, using a series of SMA springs, can provide the field perturbation required to compensate for the effects of temperature drift. Also, since this design is thermally actuated, there is no added power consumption, a very desirable property in space applications. It is then possible to design a lighter resonator out of aluminum, and compensate for the undesirable temperature drift using this SMA-actuated tuning rod. The added mass of the tuning-rod assembly, when compared to the mass savings of using aluminum, is negligible.

In order to complete this design, expressions for the effect of thermal expansion on a resonator must be derived. These expressions, along with a field perturbation model, will be used to form a sound basis for the mechanical design.

The first step in this design is to develop a model for the uncompensated temperature response of a conductor-loaded cavity (Section III). Using this model in conjunction with a field perturbation model, an expression for the resonant frequency of an SMA-compensated resonator can be derived (Section IV). A prototype was built for an SMA-compensated resonator. The temperature response of the prototype resonator was determined experimentally (Section V). These results were used to validate the model for the SMA-compensated resonator (Section VI). Finally, these results are used to draw conclusions about the general behavior of SMA-compensated resonators (Section VII). They are also used to provide a procedure for specifying alloys used to build the SMA actuator.

II. SMA Behavior

SMAs are materials that can revert to a memorized shape when heated above some threshold. This shape change is the result of a phase transformation from martensite to austenite. Annealing the alloy in a constrained shape sets the memorized shape. If an SMA compression spring is constructed, and some bias force is applied (using a conventional spring or a mass as a load), the spring will compress at low temperature. As the SMA is heated, it will transform to the stiffer austenite phase. In attempting to revert to its original uncompressed shape, it will lengthen, exerting a force on the load.

Fig. 1 shows the temperature response of an SMA wire with an applied load. The most important aspect of the SMA response from a design perspective is the hysteresis. Actuation will occur

Manuscript received April 17, 2003.

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Digital Object Identifier 10.1109/TMTT.2003.820166

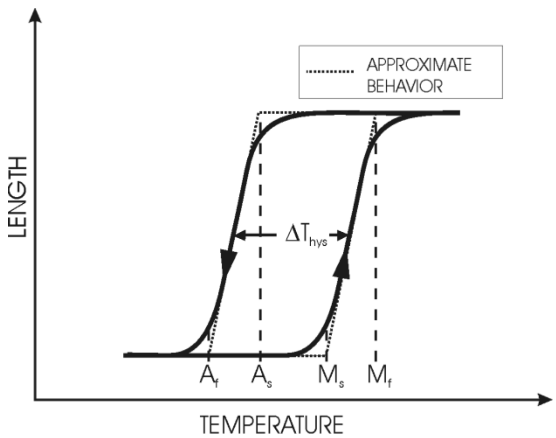


Fig. 1. SMA spring behavior.

at a higher temperature than relaxation. The actuation and relaxation response is parallel. The hysteresis of an SMA can be quantified as the temperature offset of the actuation and relaxation response curves. The actuation temperature and hysteresis of an SMA alloy are determined by the composition of the alloy. Actuation temperature can be tailored in the range of -200 °C to greater than 100 °C. SMA suppliers can tailor these actuation temperatures with a precision of ± 5 °C. Hysteresis values for particular alloys can vary from 10 °C to 100 °C.

III. Modeling of Temperature Drift

To derive a relationship between the resonant frequency of a cavity and temperature, a relationship between temperature and geometry must be established. For metals, which are allowed to expand freely, linear expansion can be assumed. The fractional expansion is $x\,y$ and z is

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha \Delta T \tag{1}$$

where α is the CTE and ΔT is the temperature change. The volume will expand, and the shear strain will be zero [5]. For a specific application, the unconstrained expansion assumption must be evaluated on a case-by-case basis. Factors affecting this assumption will include temperature range and mounting.

Assuming free expansion for a conductor-loaded rectangular waveguide cavity resonator with dimensions $b < a < d$, it can easily be shown that the resonant frequency after a rise in temperature ΔT is given by

$$f(\Delta T) = \frac{c\sqrt{a_o^2(1+\alpha\Delta T)^2 + d_o^2(1+\alpha\Delta T)^2}}{2a_o d_o (1+\alpha\Delta T)^2} \tag{2}$$

$$f(\Delta T) = f_o \frac{1}{(1+\alpha\Delta T)} \tag{3}$$

where f_o is the resonant frequency of the unperturbed resonator.

Since we are dealing with an extremely small shift in resonator frequency, it is worthwhile to check the accuracy of the High Frequency Structure Simulator (HFSS) solution. The HFSS simulated temperature drift results correlate extremely well with the analytical results in (3). The largest error in the predicted results is 0.0008%. This comparison is shown in Fig. 2.

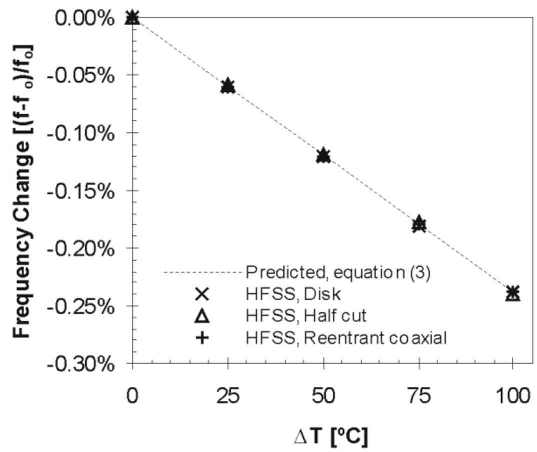


Fig. 2. Resonant-frequency predictions.

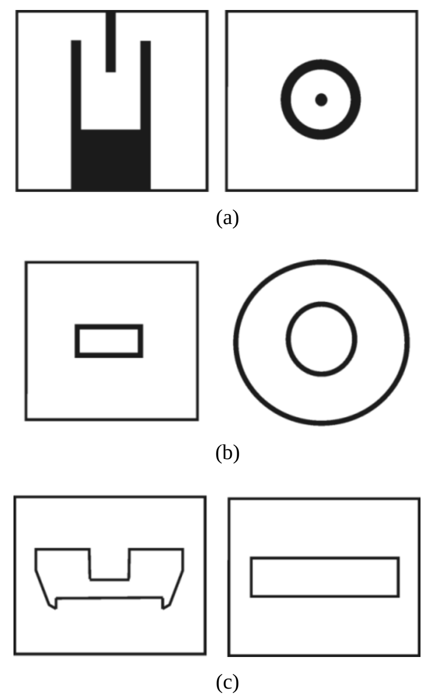


Fig. 3. (a) Reentrant coaxial resonator [6]. (b) Disk resonator [6]. (c) Half-cut resonator [8].

Simulations using HFSS suggest that this formula is valid for a conductor-loaded cavity of any shape. Equation (3) was tested using a reentrant coaxial resonator [see Fig. 3(a)] [6] a metal disk resonator [see Fig. 3(b)] [7], and a half-cut resonator [see Fig. 3(c)] [8]. No support structure was included in these simulations for simplicity.

The results of these simulations are shown in Fig. 2. Again, the HFSS simulations conform to (3) extremely well. Here, the maximum error incurred is 0.0014%.

The high correlation between the simulated and predicted resonant frequencies for these resonators implies that the temperature model in (3) is valid for any conductor-loaded cavity that is allowed to expand freely regardless of its shape.

IV. Compensated Resonator Behavior

In order to derive the compensated temperature response of the resonator, the uncompensated temperature model of the resonator (3) and a model for the field perturbation must be combined.

A modified version of the small perturbation model for a rectangular cavity is used [9].

$$\frac{f - f_o}{f_o} = -s\frac{\Delta V}{V_o}. \tag{4}$$

The parameter s must be fit to simulated data for a particular resonator ($s = 2$ for a rectangular cavity resonator). This model assumes that the perturbation response is linear, which is a reasonable assumption for the range of perturbation required for this application.

In (4), the base frequency f_o refers to the resonant frequency of the unperturbed cavity. If the cavity is subject to temperature change, this frequency will change according to (3).

The term V_o refers to the volume of the unperturbed cavity. This will change as the resonator is subject to temperature change in accordance with (1).

If these results are substituted into the modified perturbation model (4), the resonant frequency of a cavity subjected to temperature changes and field perturbations can be written as

$$f = f_o\frac{1}{(1+\alpha\Delta T)}\left(1 - s\frac{\Delta V}{V_o(1+\alpha\Delta T)^3}\right). \tag{5}$$

In (5), the base frequency f_o corresponds to the resonant frequency of the cavity with no temperature change, and unperturbed by the tuning rod.

In order to use (5) for a cylindrical tuning rod, the perturbation volume ΔV must be written in terms of the tuning-rod radius r and length $l(\Delta T)$. The cavity will be tuned using a tuning rod attached to an SMA actuator. The length of the tuning rod $l(\Delta T)$ is dependent on temperature, while the radius r is constant as follows:

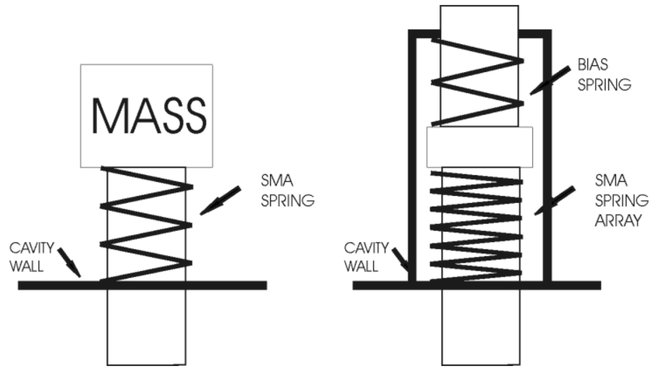
$$f = f_o\frac{1}{(1+\alpha\Delta T)}\left(1 - s\frac{\pi \cdot r^2 \cdot l(\Delta T)}{V_o(1+\alpha\Delta T)^3}\right). \tag{6}$$

V. Tuning Rod Design

Increasing the temperature of a conductor-loaded cavity decreases the resonant frequency (3). In order to counteract the effect of temperature change, the length of the tuning rod must decrease as temperature increases. This will, in turn, require an actuator that retracts as temperature increases.

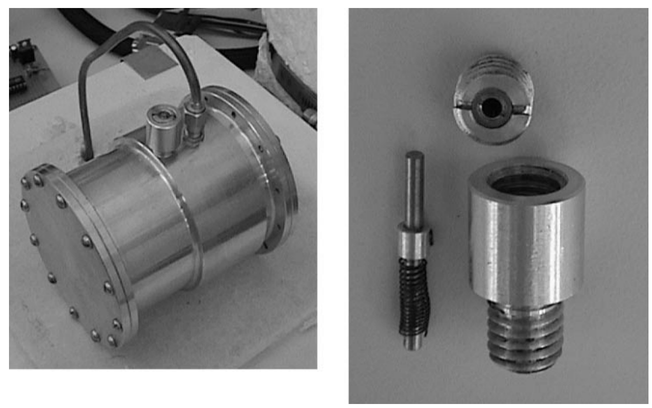
A compression spring design was chosen for this application. From Fig. 1, it can be seen that an SMA compression spring will expand as temperature increases. In order to translate this into the desired response of a decrease in tuning-rod length for an increase in temperature, the compression spring must be placed outside the cavity.

In order to design this actuator with an SMA, a bias force must be provided. A bias spring is used for this purpose [see Fig. 4(b)]. In order to simplify the test rig, a mass-biased design was used for testing [see Fig. 4(a)].



(a) (b)

Fig. 4. (a) Test design. (b) Full tuning-rod design.



(a) (b)

Fig. 5. (a) Cavity with SMA actuator and RF probe attached. (b) Assembly view of the SMA actuator.

To provide a fixed end for the bias spring, a housing is required. A prototype of this design was constructed, along with an aluminum resonator, as shown in Fig. 5. The tuning rod shown in Fig. 5(b) has a 2-mm diameter.

When specifying the SMA and bias springs for a real application, it is important to consider the possible effects of vibration. These effects will usually be quite small since the natural vibration frequency of the assembly is high. Using thicker wires for both the bias and SMA springs will increase the natural frequency of the tuning-rod assembly and reduce the magnitude of vibration when the natural frequency is excited.

An SMA spring was formed using nickel titanium wire. The SMA wire, supplied by Mondo Tronics Inc., San Rafael, CA, had a diameter of 250 μm. A mass bias of 29 g was used for these experiments. The measured actuation start A_s was 67 °C. The actuation finish A_f was 77 °C. The actuator started relaxing at a martensite start M_s of 47 °C, and finished relaxation at a martensite finish M_f of 37 °C. The response is shown in Fig. 6.

The actuator was designed so that when the SMA spring was fully actuated, the tuning-rod penetration would be zero. Using (6), the behavior of the compensated resonator can be predicted for this actuator.

The predicted resonant frequency of the SMA-compensated resonator is shown in Fig. 7. Outside of the SMA actuation range (from 0 °C to 60 °C on heating, and from 95 °C to 43°C on cooling), the resonator shows the same linear frequency drift expected from an uncompensated resonator, given by (3).

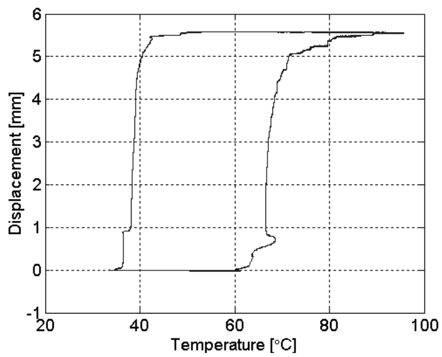


Fig. 6. SMA actuator response.

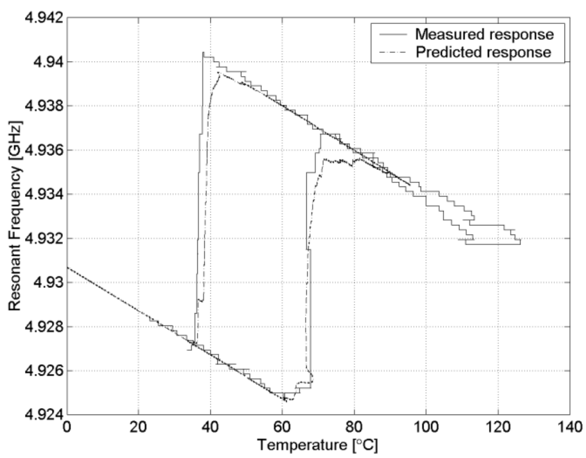


Fig. 7. SMA-compensated resonator response.

Over the actuation range of the SMA (60 °C–80 °C on heating, 43 °C–35 °C on cooling), the resonant frequency is modified considerably by the change in tuning-rod length. This change is counteracted by the reduction in the resonant frequency as the cavity expands. With alloys that actuate over a larger temperature range, the slopes resulting from tuning-rod length and cavity expansion would be better matched. This would result in better temperature compensation.

VI. Experimental Results

In order to prove that the concept of temperature compensation using SMAs is feasible, some experiments were performed. A mass-biased tuning rod was designed [see Fig. 4(a)]. Using a mass bias simplified the experimental setup considerably.

Using the actuator temperature response, the expected resonant frequency can be calculated using (6). This predicted result is compared with measured resonant frequency in Fig. 7.

Fig. 7 shows good correlation between predicted and measured results.

VII. Analysis and Design Method

The prototype SMA-actuated tuning rod has a total stroke of 5.5 mm, a perturbation that corresponds to an increase in resonant frequency of 12.55 MHz. This change in frequency is referred to as Δf_{act}. The size of the temperature range over which

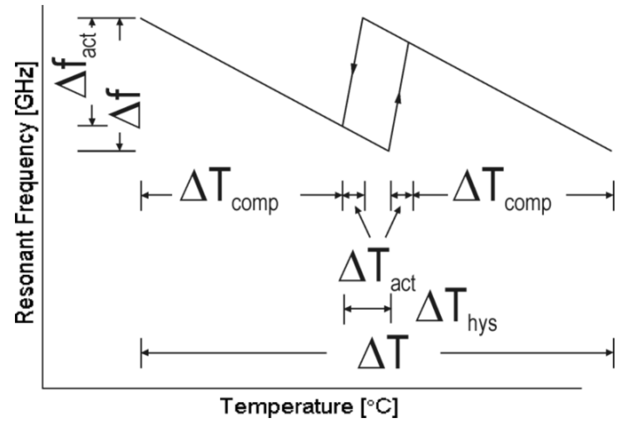


Fig. 8. Generalized compensated response, one alloy SMA-compensated resonator.

alloy actuation takes place is referred to as ΔT_{act} (Fig. 8); in this case, 10 °C. There are two factors contributing to this change in frequency in ΔT_{act}: the increase in resonant frequency caused by reducing the length of the tuning rod with the actuator, and the decrease in resonant frequency caused by the expansion of the cavity. Since the tuning-rod perturbation is the dominant effect (6), an increase in the actuation range ΔT_{act} results in a decrease in Δf_{act}. This reduction will reduce the overall temperature drift.

If the tuning rod increases the resonant frequency by Δf_{act}, this will compensate for a decrease in frequency due to temperature drift. From (3),

$$\Delta f_{\mathrm{act}} = -\Delta f_{\mathrm{drift}}$$
$$\Delta T = \left(\frac{1}{f(\Delta T)}\right)\frac{f_o - f(\Delta T)}{\alpha}$$

assume that

$$\frac{1}{f(\Delta T)} \cong \frac{1}{f_o}$$
$$\Delta T_{\mathrm{comp}} = \frac{\Delta f_{\mathrm{act}}}{\alpha \cdot f_o}. \tag{7}$$

The resonant frequency of the cavity drifts normally over ΔT_{comp}. The SMA-actuated tuning rod compensates for this drift over ΔT_{act}. If the actuator hysteresis is given by ΔT_{hys}, then the total temperature range (ΔT) is given by

$$\Delta T = 2 \cdot \Delta T_{\mathrm{comp}} + \Delta T_{\mathrm{act}} + \Delta T_{\mathrm{hys}}. \tag{8}$$

Over the temperature range ΔT, the resonator drifts over the equivalent of $\Delta T_{\mathrm{hys}} + \Delta T_{\mathrm{comp}}$. That is to say that the frequency range of the compensated resonator is the same as that of an uncompensated resonator subjected to a temperature change of $\Delta T_{\mathrm{hys}} + \Delta T_{\mathrm{comp}}$. From (3),

$$\Delta f = \frac{f_o \alpha(\Delta T_{\mathrm{hys}} + \Delta T_{\mathrm{comp}})}{1 + \alpha(\Delta T_{\mathrm{hys}} + \Delta T_{\mathrm{comp}})}. \tag{9}$$

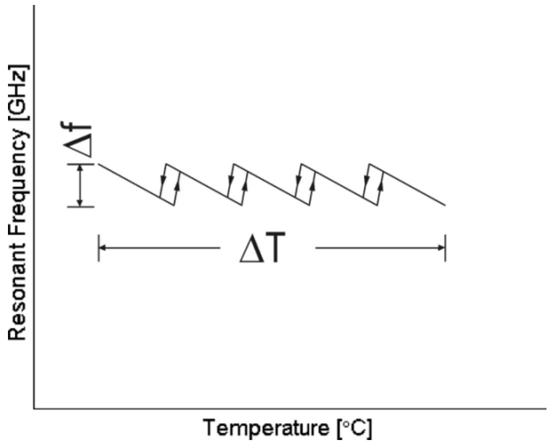


Fig. 9. Generalized compensated response, four-alloy SMA-compensated resonator.

The temperature drift over the entire temperature range can now be calculated using (8) and (9) as follows:

$$\delta = \frac{\Delta f}{\Delta T_m f_d 10^{-6}}$$
$$\delta = \frac{\alpha(\Delta T_{\rm hys} + \Delta T_{\rm comp})10^6}{\left(1+\alpha(\Delta T_{\rm hys}+\Delta T_{\rm comp})\right)\Delta T}. \tag{10}$$

The SMA spring used for testing shows a hysteresis of $\Delta T_{\rm hys} = 29$ °C and $\Delta T_{\rm act} = 10$ °C. The tuning rod increases the resonant frequency by $\Delta f_{\rm act} = 12.55$ MHz. For a desired resonant frequency, $f_o = 4.94$ GHz. The expected temperature drift can be calculated.

The calculated drift using (10) is 10.7 ppm/ °C. Experimental results from Fig. 7 can be used to confirm this result. When the temperature drift is extrapolated to the limit temperatures, the temperature drift is 10.9 ppm/°C. An uncompensated aluminum resonator would show a temperature drift of 20 ppm/°C.

Since this tuning rod has a stroke of 5.5 mm, the increase in the resonant frequency due to actuation is large. Decreasing the size of the SMA spring easily changes this. By reducing the stroke of the tuning rod, the compensated temperature range ΔT is reduced, as well as the frequency change Δf. This results in a small increase in frequency drift, but brings the compensated temperature range down to a more reasonable level (currently $\Delta T = 286$ °C.)

An ideal material might have a hysteresis of 10 °C and an actuation range of 10 °C. The resulting tuning-rod actuator could have a $\Delta f_{\rm act} = 500$ kHz. This would result in a temperature drift of 10 ppm/°C over a temperature range of 30 °C.

Since the compensated temperature range has been reduced to 30 °C, using a series of alloys can further reduce the temperature drift. By building the actuator using multiple SMA springs in series, the resulting perturbation can be spread out piecewise over a wider temperature rage. The resulting response is shown in Fig. 9.

For an actuator with n alloys, the new temperature change ΔT is given by

$$\Delta T = n(\Delta T_{\rm comp} + \Delta T_{\rm act} + \Delta T_{\rm hys}) + \Delta T_{\rm comp}$$

If four SMA alloys in series are used to design the tuning-rod actuator, the increase in temperature range results in a temperature drift of 2.9 ppm/ °C. This is a substantial reduction of the uncompensated temperature drift of 20 ppm/°C.

VIII. Conclusion

This paper has presented an analytically derived model for the temperature drift of conductor-loaded cavity resonators shown in (3). It can be concluded that this expression is valid for any conductor-loaded cavity resonator regardless of its shape. This result was used to form a comprehensive method for designing SMA tuning-rod actuators capable of compensating for temperature drift. An expression was given for the design performance in terms of temperature drift. This expression was confirmed using a single alloy tuning-rod actuator that reduced the temperature drift in an aluminum resonator from 20 ppm/°C to 10 ppm/°C. A design was proposed for a multiple-alloy tuning-rod actuator that would further reduce the temperature drift to 2.9 ppm/°C given an appropriate material. This SMA temperature-compensation design is feasible and can substantially reduce the temperature drift of conductor-loaded cavities. Furthermore, these design techniques can be used to easily calculate the performance of a tuning-rod actuator. When compared with alternative materials such as invar, the use of aluminum resonators offers significant mass savings. In mass critical designs, such as satellite applications, this mass savings is extremely valuable.

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